Canonical Description of Strangeness Enhancement from p-A to Pb-Pb Collisions

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Abstract

We consider the production of strange particles in Pb-Pb and p-A collisions at the SPS energy reported by the WA97 experiment. We show that the observed enhancement of strange baryon and antibaryon yields in Pb-Pb collisions relative to p-Be and p-Pb can be explained in terms of the statistical model formulated in canonical ensemble with respect to strangeness conservation. The importance and the role of strangeness under saturation is also discussed.

1 Introduction

Various experiments at CERN SPS reported an enhancement of strange particle production when going from p-p, p-A to A-A collisions [1, 2, 3, 4, 5]. In particular the recent analysis of Pb-Pb reaction by the WA97 collaboration has shown, by comparing p-Be, p-Pb with Pb-Pb, a strong enhancement in the pertinent strange and antistrange baryon yields [2, 3, 4]. In addition this enhancement was found to increase with strangeness content of the particle.

In this letter we show that the observed features of the data can be well described by the thermal model for particle production using canonical formulation of strangeness conservation.

The statistical models formulated in the grand canonical (GC) ensemble, with respect to conservation laws, were shown to give a satisfactory description of particle production in heavy ion collisions at SPS and AGS energies [6, 7, 8, 9, 10]. However, when applying this model to the low energy heavy ion [11], or high energy hadron-hadron or e^+e^- reactions one necessarily needs to treat the conservation laws exactly [12]. The exact conservation of quantum numbers, that is the canonical (C) approach, is known to reduce the phase space available for particle production due to additional constraints appearing through requirements of local quantum number conservation [13, 14, 15, 16]. Thus, the immediate

question is to what extend the canonical suppression can account for the observed effect in the WA97 experiment.

The concept of canonical suppression as an origin of strangeness enhancement has been already addressed in the literature [17] and also tested in the context of the SPS data [18]. Indeed, it was shown [18] that the increase of K^0_S/h^- and $(\Lambda+K)/\pi$ ratios from p-p to A-A collisions can be possibly explained by the canonical effect. In view of the new data from the WA97 collaboration we will test further this idea and show that the suppression of strange particles phase space when going from GC to C formulation of strangeness conservation can quantitatively explain the reported data.

In the following section we present the main aspects of the statistical model formulated in canonical ensemble and then we compare the model with the experimental data.

2 Canonical Formulation of Strangeness Conservation

We consider a gas composed of all particles and resonances with mass up to $m \simeq 2.4$GeV. Conservation of baryon number is introduced in GC ensemble, thus it is controlled by baryochemical potential. Strangeness conservation, however, is introduced on the canonical level. Having in mind the application of the model to particle production in heavy ion collisions we also require that the overall strangeness of the system is zero. Using the general procedure for the canonical strangeness conservation [15, 16] the partition function of a gas of particles with strangeness $0, \pm1, \pm2, \pm3$ can be written as follows:

$$Z_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \;\exp\,(\sum_{s=-3}^{3} S_s e^{is\phi}), \tag{1}$$

where $S_s = V_0 \sum_i Z_i$ and the sum is taken over all particles and resonances carrying strangeness s. For particle of mass m_i, degeneracy factor d_i, baryon number B_i, and assuming Boltzmann statistics we have:

$$Z_i = \frac{d_i}{2\pi^2} m_i^2 T K_2(m_i/T)\exp\,(B_i\mu/T). \tag{2}$$

With the particular form of the partition function given by Eq. (1) the density n_s of particle i with strangeness s in volume V is obtained by the replacement $Z_i \mapsto \lambda_i Z_i$ in Eq. (1) and then taking an appropriate derivative [15, 16]:

$$\begin{aligned} n_{\pm s} \;&=\; \frac{\langle N_{\pm s}\rangle}{V} \equiv \frac{\lambda_i}{V_0}\frac{\partial \ln Z_0}{\partial\lambda_i}|_{\lambda_i=1} \simeq Z_{\pm s}\frac{(S_{\pm1})^s}{(S_{+1}S_{-1})^{s/2}} \\ &\{I_s(x_1)I_0(x_2) + \sum_{m=1}^{\infty} I_m(x_2)[I_{2m+|s|}(x_1)A^{m/2} + I_{2m-|s|}(x_1)A^{-m/2}]\}/Z_0 \end{aligned} \tag{3}$$

where

$$x_k \equiv 2\sqrt{S_k S_{-k}} \;\;,\;\; A \equiv \frac{S_{-1}^2 S_2}{S_1^2 S_{-2}}, \tag{4}$$

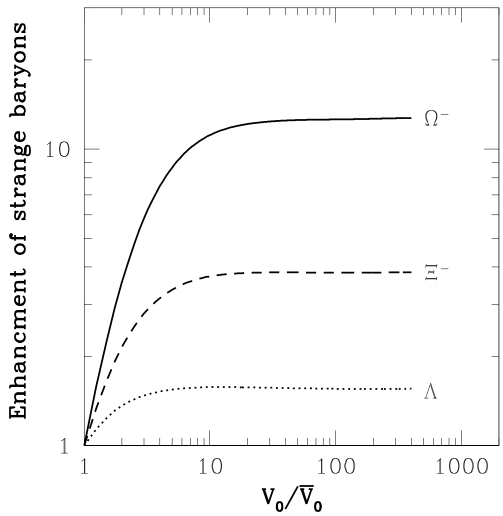

Figure 1: Volume dependence of strange baryon densities normalized to their value calculated in $V_0 = \bar{V}_0 = 7.4$ fm^3 for $T = 168$ MeV and $\mu = 266$ MeV.

and the partition function

$$Z_0 \simeq I_0(x_1) I_0(x_2) + \sum_{m=1}^{\infty} I_{2m}(x_1) I_m(x_2) [A^{m/2} + A^{-m/2}]. \tag{5}$$

In the derivation of Eq. (3) we have neglected, after differentiation over particle fugacity, the term $S_{\pm 3}$. This approximation, however, due to small value of $S_{\pm 3}$ coefficients in comparison with $S_{\pm 1}$ and $S_{\pm 2}$ is quite satisfactory. The complete expression for the partition function without any approximation can be found in reference [16].

Due to the locality of strangeness conservation, we have introduced two volume parameters, V and V_0 in Eq. (3) [13]. The parameter V describes the volume of the system at freeze out, thus V is also the overall normalization factor which fixes the particle multiplicity from the corresponding density. The volume V_0 is interpreted as a correlation volume where particles are created to fulfill the requirement of local conservation of quantum numbers. It was shown [11] that in the canonical description of particle production in low energy heavy ion collisions $V \neq V_0$ is required to reproduce both the particle multiplicity ratios and the particle yields. The volume parameter V scales with the total number of participants, $V \sim A_{part}$, whereas $V_0 = 4\pi R_0^3/3$ where $R_0 \simeq (1.15 \pm 0.05) A_p^{1/3}$ with A_p being the number of the projectile participants.

Comparing the result of Eq. (3) with the standard expression for the particle density in GC ensemble, one can see that the term multiplying $Z_{\pm s}$ in Eq. (3) plays the role of the strangeness fugacity [11, 15]. Indeed taking the limit for large x_i, in Eq. (3), that is the limit of high temperature and/or large V_0, the GC result for the particle density is recovered. In the opposite limit the C ensemble leads to a strong suppression of strange particle densities.

In Fig.1 we show the dependence of the density of strange particles on V_0 for fixed $T = 168$ MeV and $\mu = 266$ MeV. In order to illustrate the enhancement with increasing

volume and strangeness content of the particle we show in Fig.1 the density of Λ, Ξ^- and Ω baryons normalized to its value calculated in the volume $V_0 = \bar{V}_0 = 7.4$ fm^3, as a function of $V_0/\bar{V}_0$. It is clear from Fig.1 that the particle densities are increasing with the size of the system. For sufficiently large V_0 the densities are saturating which indicates that the GC limit is reached. The enhancement is seen to increase with the strangeness content of the particle and obviously depends on the choice of $\bar{V}_0$. Keeping the value of baryon chemical potential and/or temperature as being V_0 independent, the enhancement of antibaryons is similar to the one seen in Fig.1 for baryons. Thus, the possible asymmetry in enhancement for strange baryons and antibaryons when going from small to large system requires μ and/or T to be V_0 dependent.

In the following section we will discuss to what extend the above model can be used to understand strangeness enhancement versus centrality for baryon and antibaryons reported by the WA97 collaboration.

3 Comparison with the Experimental Data

It is seen in Fig.1 that the model reproduce the main features of centrality dependence of strangeness enhancement in the WA97 data. The quantitative comparison of the model with the experimental data require, however, a particular choice of the parameters, that is: the temperature T, the volume V_0 and the baryon chemical potential μ. We have used for Pb-Pb collisions: $T = 168$ MeV and $\mu = 266$ MeV, the values which have been shown [6] to give a good description of all data on particle multiplicity ratios measured by the NA49 and WA97 experiments. In general temperature and chemical potential could vary with V_0 that is also with A_p. From previous analysis we know [19] that for the same collision energy the temperature variations with A_{part} is small. Thus, we keep T to be the same for both, p-Be and Pb-Pb systems. The chemical potential, however, as discussed in the previous section, has to be taken as A_{part} dependent since the data are showing larger enhancement for strange than for antistrange baryons. This indicates lower value of μ in p-Be than in Pb-Pb system. We use $\bar{\mu}$ in p-Be as a free parameter which is chosen to reproduce the enhancement asymmetry between Ξ^- and $\bar{\Xi}^+$ or simply the $\Xi^-/\bar{\Xi}^+$ ratio measured in p-Be collisions.

Following the interpretation of V_0, described in the previous section, we chose $R_0 \sim 1.2$ fm in p-Be that is $V_0 = \bar{V}_0$. To make a quantitative comparison of the model with the experimental data we are left with only one parameter, the effective chemical potential $\bar{\mu}$ in p-Be which was taken to be 150 MeV. This value is required in comparison with $\mu = 266$ MeV in Pb-Pb in order to obtain the correct splitting between baryons and antibaryons of the enhancement factors without affecting the average enhancement factors.

In Fig.2 we show the densities calculated in C ensemble for large V_0 normalized to its value at $\bar{V}_0$ in to p-Be system. This corresponds to the experimental comparison of yield/participant of Pb-Pb to p-Be system. The full lines in Fig.2 are calculated from Eq.(3) by varying V_0 and keeping $\mu = 266$ MeV. The change of V_0 with centrality in Pb-Pb collisions is included relating $V_0 = 4\pi R_0^3/3$ with A_{part} through, $R_0 \sim 1.2(A_{part}/2)^{1/3}$, which determines approximately the number of projectile participants.

The actual path from grand canonical to canonical result for the particle densities

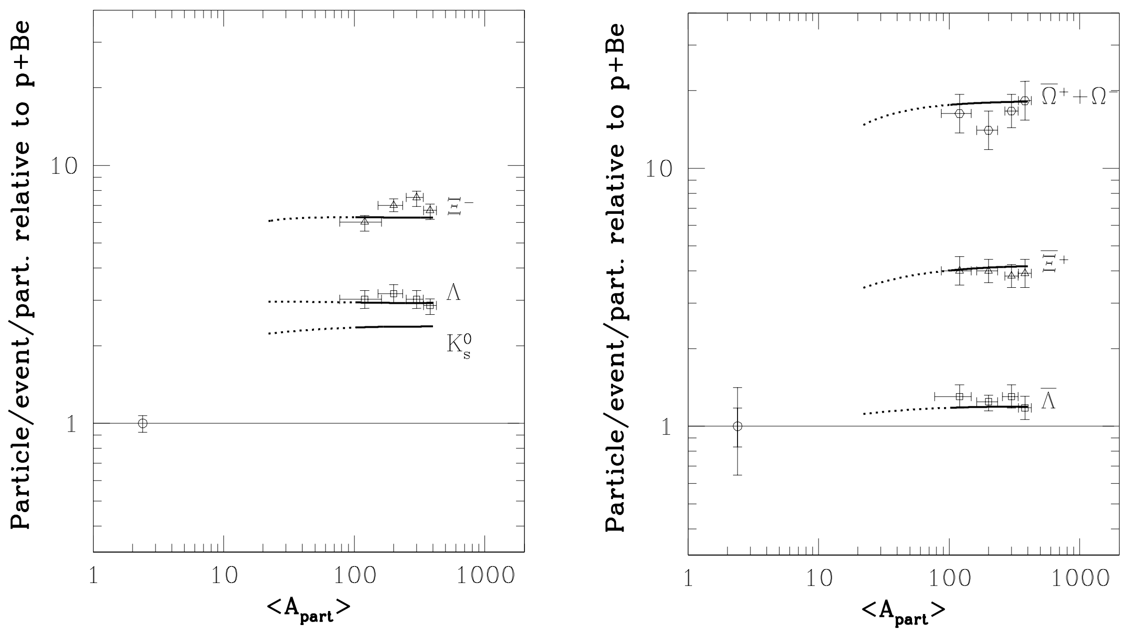

Figure 2: Particle yields/participant for strange baryons and antibaryons calculated in the canonical ensemble with $V_0 \sim A_{part}/2$ normalized to their values at $\bar{V}_0$. Also shown are the WA97 experimental data for yields per participant in Pb-Pb relative to p-Be [2]. For description of curves see text.

would required an additional knowledge of A_{part} dependence of chemical potential which is effectively changing from 266 MeV in Pb-Pb to 150 MeV in p-Be system. Formulating, in addition to strangeness, also baryon number conservation canonically, the path from GC to C result as a function of V_0 can be established [20]. In the simplified model, presented here, where baryon number conservation was introduced effectively trough chemical potential we can only study the relative enhancement of strange baryon densities from small p-Be to large Pb-Pb system.

The result presented in Fig.2 shows that the relative enhancement of strange baryon and antibaryon yield/participant from p-Be to Pb-Pb, found experimentally, can be well described by the effect of canonical suppression and the change of μ from 266 MeV in Pb-Pb to 150 MeV in p-Be collisions. The enhancement appears in the model to be almost saturated for $A_{part} \simeq 100$ which indicates that the system is close to its GC limit. The enhancement pattern for different strange baryons and antibaryons densities is also understood within the model as a consequence of different dependence on V_0. In the large system like Pb-Pb, and for large collision energy required to maintain high T, the density n_s of particle carrying strangeness s, is V_0 independent. In the opposite limit, however, this dependence is changed to $n_s \sim V_0^s$ which can be verified from Eq. (3). For small x_i we have approximately:

$$n_{\pm s} \simeq Z_{\pm s} \frac{(S_{\pm 1})^s}{(S_{+1}S_{-1})^{s/2}} \, \frac{I_s(x_1)}{I_0(x_1)}. \tag{6}$$

Expanding the Bessel functions $I_n(x) \sim x^n$ in Eq. (6) we see that $n_s \sim V_0^s$.

The variation of V_0 with the number of projectile participants and the assumption that $V \neq V_0$ is also justified by the data. Indeed the recent data of the WA97 collaboration show that strange particle yield/A_{part} is independent on A_{part} in p-A collisions. This is well reproduced by canonical treatment since V_0 is the same for all A in p-A collisions. If one could, however, take $V = V_0$ then, since $V \sim A_{part}$, a strong variation with A_{part} of strange particle densities should be observed in p-A collisions when changing A. In addition this variation would depends on the strangeness content of the particle which is not experimentally observed in p-A. Thus from the point of view of the considered canonical model, p-Be and p-Pb are the same system with the only difference being contained in slightly larger $\bar{\mu}$ in p-Pb then p-Be. We found that the chemical potential in p-Pb increases by about 10% with respect to p-Be. This explains similar excess of particle yield/A_{part} in Pb-Pb relative to p-Be and p-Pb collisions.

Until now we have assumed that p-A as well as Pb-Pb system can be described by the statistical model in chemical equilibrium. For Pb-Pb collisions chemical equilibrium assumption has been found to give a good agreement of the model with the experimental data [6]. In p-A collisions, however, due to small time scale, deviations from strangeness chemical saturation cannot be excluded. In a previous analysis [12] of particle production in $\mathrm{p}-\mathrm{p}$, $\mathrm{p}-\bar{\mathrm{p}}$ and $e^+ - e^-$ collisions in terms of C ensemble strangeness under saturation was accounted for by introducing an additional parameter γ which measures deviation of strangeness from full equilibrium [21]. Also in this approach V_0 was assumed to coincide with V. In the following we discuss how the possible out of chemical equilibrium together with the $V = V_0 \sim A_{part}$ assumption will modify our results and conclusions.

To introduce non-equilibrium effects via γ parameter is quite straightforward [21, 10]. It amounts to the replacement $S_{\pm s} \mapsto \gamma^s S_{\pm s}$ in Eq. (3). First we observe that applying the limit $x_i \mapsto 0$ in Eq. (6) the density n_s of particle with strangeness s, scales as $(\gamma^2 V)^s$. For larger x_i we found that, in the parameter range relevant for p-A system, this dependence is changed approximately to $n_s \sim (\gamma^{2.7} V)^s$. Thus, keeping $\gamma^{2.7} V = \bar{V}_0$, the value used in equilibrium model, we can as well reproduce the experimental data shown in Fig.2 in non-equilibrium formulation. In this approach, however, in order to reproduce experimentally observed linear dependence of strange particle yields with A_{part} in p-A collisions [2], one needs, for each value of A_{part}, to adjust γ such that $\gamma^{2.7} A_{part} = const.$ Obviously, this requires a decreasing γ with A_{part}. In general, one would rather expect the opposite behavior. In order to ovoid, however, the possible ambiguity with respect to strangeness undersaturation factor γ one needs to perform an additional detailed analysis in proton induced collisions for different targets and collision energies. This analysis could possibly settle to what extend there is a need for genuine global strangeness enhancement related with the change in γ between p-A and A-B collisions.

4 Summary

We have used the comprehensive set of the recent results of the WA97 collaboration on the production of strange and multistrange hadrons in p-A and Pb-Pb collisions in order to check the validity of statistical model description of strangeness enhancement centrality dependence. The statistical model was verified to give a good agreement with all

experimental data in A-A collisions at SPS energies. From the point of view of statistical mechanics the application of the model for smaller system like p-A requires obvious modifications. In the large system strangeness is conserved on the average, thus it is controlled by strange chemical potential in the grand canonical ensemble. In the small system, however, strangeness conservation laws must strictly be obeyed in each single collision which requires canonical ensemble. We have shown that the required passage from grand canonical to canonical ensemble reproduces the basic features of strangeness suppression from Pb-Pb to p-Be or p-Pb collisions reported by WA97 collaboration. The observed asymmetry in the enhancement of strange baryons and antibaryons requires additional assumption on A_{part} dependence of baryon chemical potential μ. We have shown that with the particular choice of μ asymmetry in p-A and Pb-Pb systems the model provides good quantitative description of the data.

Deviation from chemical equilibrium has been included in our discussion only in p-A system. Recently an interesting non-equilibrium model for the description of particle production in A-B collisions has been proposed [22]. The interpretation of centrality dependence of strangeness enhancement in a consistent non-equilibrium picture would require additional analysis not presented in this paper.

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